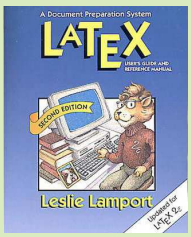
NORMANHURST BOYS HIGH SCHOOL

MATHEMATICS DEPARTMENT

Topic summary and exercises:

- LaTeX for teaching

With references to



Name: .....

Initial version by H. Lam, October 2014. Last updated September 14, 2021.
Various corrections by students and members of the Department of Mathematics.

Acknowledgements Pictograms in this document are a derivative of the work originally by Freepik at `http://www.flaticon.com`, used under CC BY 2.0.

Symbols used

(!) Beware! Heed warning.

Gentle reminder

- Practise makes perfect! Use `https://overleaf.com` or `http://texrendr.com`.
- Get help by asking on Teams/searching Google.

Contents

Section 1

Introduction

1.1 Why use LaTeX when we have Word?

1. Industry standard.[1]
2. Gaining greater acceptance for mathematics input in online forums & social media (e.g. Edmodo, Moodle, Canvas)
3. (Eventually) less time spent on formatting, more time on getting the content right.
4. Free! (In general)

```
\section{Why use \LaTeX\ when we have Word?}

\begin{enumerate}
\item   Industry standard.\footnote{Outside
    of the world of mathematics teaching in
    NSW secondary schools.}
\item   Gaining greater acceptance for
    mathematics input in online forums \&
    social media (e.g. Edmodo, Moodle,
    Canvas)
\item   (Eventually) less time spent on
    formatting, more time on getting the
    content right.
```

Figure 1.1 – Part of this document under construction

1.2 What is (and is not) LaTeX

1.2.1 LaTeX is ...

- A *typesetting* system, which contains a collection of programs, scripts and packages.
- Open source and *free* – not subject to any one company maintaining the software up to date, but rather motivated individuals and organisations.
- entirely *plain text* code, i.e. human readable source files, and can be read by simple applications such as *Notepad*.

1.2.2 LaTeX is *not* ...

- A *word processor* (e.g. Microsoft Word).
- A single program.
- Necessarily easy to start learning. *However* once you wrap your mind around it, it becomes significantly easier.

[1] Outside of the world of mathematics teaching in NSW secondary schools.

Part I

Basic math input with LaTeX

Section 2

Via texrendr.com

`texrendr.com` is a simple previewer for math mode content.

2.1 Partial list of LaTeX math commands/symbols

- This section is a combination of an extract from *A comprehensive list of symbols for LaTeX*, also available at the (URL) *Comprehensive TeX Archive Network* (CTAN), as well as *LaTeX– A document preparation system, User's Guide and Reference Manual*.
- Further reading is available through the (URL) *The (not so) Short Introduction to LaTeX*.

2.1.1 Symbols

α	\alpha	β	\beta	γ	\gamma	δ	\delta	ϵ	\epsilon
ε	\varepsilon	ζ	\zeta	η	\eta	θ	\theta	ι	\iota
κ	\kappa	ϑ	\vartheta	λ	\lambda	μ	\mu	ν	\nu
ξ	\xi	π	\pi	ρ	\rho	σ	\sigma	τ	\tau
υ	\upsilon	ϕ	\phi	φ	\varphi	χ	\chi	ψ	\psi
ω	\omega	Γ	\Gamma	Δ	\Delta	Θ	\Theta	Λ	\Lambda
Ξ	\Xi	Π	\Pi	Σ	\Sigma	Υ	\Upsilon	Φ	\Phi
Ψ	\Psi	Ω	\Omega						

Table 2.1 – Greek letters

$\leq$	\leq	$\geq$	\geq	$\neq$	\neq	$\approx$	\approx	$\equiv$	\equiv
$\times$	\times	$\div$	\div	$\pm$	\pm	$\mp$	\mp	$\cdot$	\cdot
$\cdots$	\cdots	$\circ$	\circ	∞	\infty	$\cup$	\cup	$\cap$	\cap
$\supset$	\supset	$\subset$	\subset	$\supseteq$	\supseteq	$\subseteq$	\subseteq	$\forall$	\forall
$\in$	\in	$\exists$	\exists	$\rightarrow$	\rightarrow	$\Rightarrow$	\Rightarrow	∂	\partial
$\therefore$	\therefore	$\because$	\because						

Table 2.2 – Some operators/relations
$\therefore$ and $\because$ requires package `amsmath`

$\angle$	\angle	$\triangle$	\triangle	$\perp$	\perp	$\parallel$	\parallel	$\|\!\|\!\|$	\interleave*

Table 2.3 – Geometry symbols
* Requires package `stmaryrd`

sin	`\sin`	cos	`\cos`	tan	`\tan`	csc	`\csc`	sec	`\sec`
cot	`\cot`	log	`\log`	$\log_{10}$	`\log_{10}`	ln	`\ln`	sinh	`\sinh`
cosh	`\cosh`	tanh	`\tanh`	gcd	`\gcd`	lim	`\lim`	max	`\max`
min	`\min`	exp	`\exp`	arg	`\arg`				

Table 2.4 – Some log-like operators

2.1.2 Subscripts and superscripts

Superscripts and subscripts can be made with the `_` (underscore) and `^` (carat) commands.

(!) Enclose grouped symbols within curly braces `{` and `}`.

(!) Failure to group with curly braces will result in strange results. *Always* check your output!

Example 1

e^{2x}	`e^{2x}`	e^{x^2}	`e^{x^2}`	x_1	`x_1`	x_{y_1}	`x_{y_1}`
w^x_y	`w^{x}_{y}`	w^x_y	`w_{y}^{x}`	$\log_{10} x$	`\log_{10} x`	$\sin^2 2x$	`\sin^2 2x`

2.1.3 Fractions

Fractions can be typeset by the `\frac` command, with two sets of curly braces to enclose the numerator and denominator:

Example 2

$\frac{x + y}{1 - xy}$ `\frac{x + y}{1 - xy}`

$1 + \frac{1}{1 + \frac{1}{1 + \cdots}}$ `1 + \frac{1}{1 + \frac{1}{1 + \cdots}}`

(!) Common error: forgetting to match up ("balance") your opening/closing curly braces.

2.1.4 Roots

The `\sqrt` command produces the radical symbol with an extensible top. It has an *optional* argument (enclosed in square brackets) to indicate other roots.

Example 3

$\sqrt{x^2 + y^2}$ `\sqrt{x^2 + y^2}`

$\sqrt[n]{a} = a^{\frac{1}{n}}$ `\sqrt[n]{a} = a^{\frac{1}{n}}`

2.1.5 Variable sized operators/delimiters

Some symbols are variable in size. Edmodo will use displayed (as opposed to inline) versions of these symbols for ease of viewing (For an explanation of Displayed vs Inline equations, see Section 3.3 on page 16).

Delimiters A *delimiter* is a symbol that acts logically like a parenthesis, with a pair of delimiters enclosing an expression. These commands and characters produce delimiters of the indicated size. However, delimiters around formulae should be big enough to "fit around" the expression that they delimit.

Important note

Use `\left`, `\right` to prefix the appropriate character/command to obtain the proper sizing.

(	(	)	)	[	[	]	]
{	\{	}	\}	⌊	\lfloor	⌋	\rfloor
⌈	\lceil	⌉	\rceil	⟨	\langle	⟩	\rangle
/	/	\	\backslash	|	|	‖	\\|
↑	\uparrow	↓	\downarrow	↕	\updownarrow	⇑	\Uparrow
⇓	\Downarrow	⇕	\Updownarrow				

Table 2.5 – Delimiters

Operators

$\sum$	\sum	$\prod$	\prod	$\int$	\int	$\binom{a}{b}$	\binom{a}{b}

Table 2.6 – Variable sized operators

To place limits on $\sum$, $\int$, square brackets and limits, use subscripts and superscripts.

Example 4

$$\sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$

```
\sum_{k = 0}^{n} \binom{n}{k} x^k y^{n - k}
```

$$\int_a^b \left(x^2 + 1\right) \: dx$$

```
\int_a^b \left(x^2 + 1\right) \: dx
```

$$\frac{d}{dx} \left(\frac{\cos^2 x}{e^{2x}}\right)$$

```
\frac{d}{dx} \left(\frac{\cos^2 x}{e^{2x}}\right)
```

$$\lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n = e$$

```
\lim_{n \rightarrow \infty}
  \left(1 + \frac{1}{n}\right)^n = e
```

Note the `\:` inside the integral is required for a small space to be inserted between the `x^2 + 1` and the `dx`.

Further exercises

In `texrendr`, typeset the following **equations** from Patel (2004, p.262-263), with properly sized delimiters. Reply to your own message to typeset the subsequent block of text.

1.

A particle of unit mass travels in a straight line against the resisting force $f(v) = v(1 + v^2)$. Its initial velocity is c m/s at the origin. Show that the time t, when velocity is v, is given by:

$$t = \frac{1}{2}\log\left[\frac{1 + v^{-2}}{1 + c^{-2}}\right].$$

Find v^2 as a function of t and hence the limiting value of v at $t \to \infty$.

2.

Prove that $v_x = U\cos\theta e^{-kt}$ and $v_y = \frac{1}{k}(g + kU\sin\theta)e^{-kt} - \frac{g}{k}$

3.

Prove: $x = \frac{U}{k}\cos\theta(1 - e^{-kt})$

$$y = \frac{(kU\sin\theta + g)}{k^2} \cdot (1 - e^{-kt}) - \frac{gt}{k}.$$

Section 3

Via Overleaf

3.1 Getting started

Prior to embarking on learning more commands/environments, please:

Steps

1. Create account at (URL) Overleaf by using your DoE email (`@det.nsw.edu.au`) as username.
2. Create a new **Blank** project under Overleaf.
3. Read the (URL) LaTeX for Word Processor Users guide.
4. Load the `amsmath` and `amssymb` packages by including `\usepackage{amsmath,amssymb}` in the preamble of the document. See Section 4.1 on page 23.

Important note

(!) Overleaf also has excellent help pages.

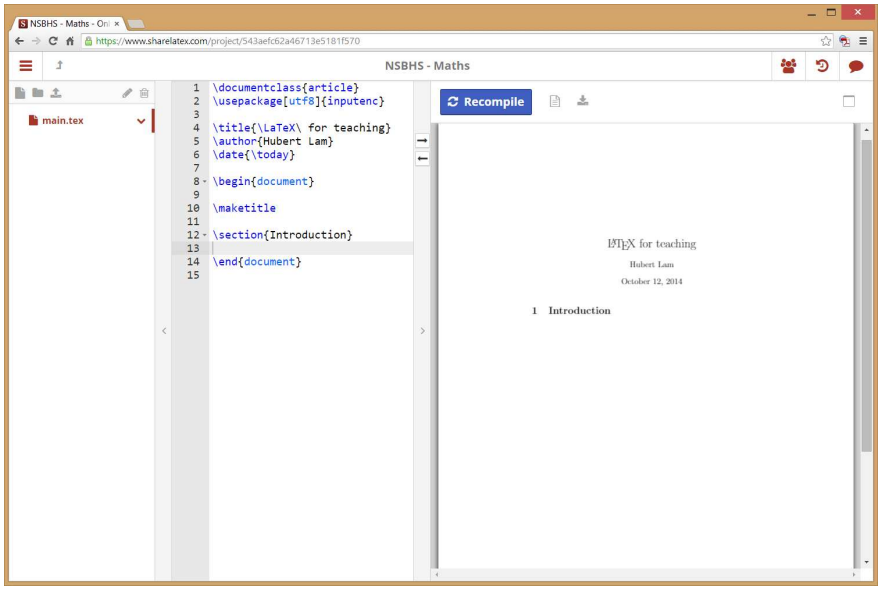


Figure 3.1 – Overleaf inside a web browser

3.2 Text mode vs Math mode

Important note

For a paragraph break in text mode, press `Enter` twice (i.e. requires two paragraph breaks)

3.2.1 Spaces, line/paragraph breaking/special symbols

Spaces/breaks

Spaces and line/paragraph breaking needs special attention in LaTeX.

Example 5

Listing 3.1 – Spacing, line/paragraph breaks

```
1 Spaces, line    and paragraph
2 breaks need to     be    dealt with carefully.
3
4 If a paragraph break is required, leave an empty line. \\
5 If only a line break is required, use a double-backslash. \\
6
7 Otherwise a double-backslash plus empty line will do the trick as well.
```

Spaces, line and paragraph breaks need to be dealt with carefully.
If a paragraph break is required, leave an empty line.
If only a line break is required, use a double-backslash.

Otherwise a double-backslash plus empty line will do the trick as well.

Special symbols

Some special symbols are normally LaTeX reserved characters. Use a backslash to typeset them:

Listing 3.2 – Special symbols

```
1 \$ \& \% \# \_ \{ \}
```

$ & % # _ { }

3.2.2 Text mode commands

Commands start with a backslash. For example, to make text bold:

Example 6

Listing 3.3 – Bold text

```
1 \textbf{Bold} text
```

Bold text

Important note

- Notice braces used to enclose argument.
- *Most* LaTeX commands are mnemonic.

Commands to alter the typeface:

- `\textbf{...}` – text boldface
- `\emph{...}` – emphasise
- `\textit{...}` – text italics
- `\texttt{...}` – text typewriter

Commands for sectioning:

- `\section{...}` – starts new section
- `\subsection{...}` – starts new subsection
- `\subsubsection{...}` – starts new subsubsection (that's it!)

Commands to alter the type size

- `\tiny`
- `\scriptsize`
- `\footnotesize`
- `\small`
- `\normalsize`
- `\large`
- `\Large`
- `\LARGE`
- `\huge`
- `\Huge`

3.2.3 Text mode environments

Environments commence with a `\begin{...}` and finish with `\end{...}`

Example 7

Listing 3.4 – Itemize environment

```
\begin{enumerate}
\item   Hello!
\item   World.
\item   Another item
        \begin{itemize}
        \item   Second level
        \item   Second level item 2
        \end{itemize}
\item   \begin{enumerate}
        \item   Nested levels
                \begin{itemize}
                \item   More levels!!!!
                \item   Even more dot points
                \end{itemize}
        \item   Nested level item 2
        \end{enumerate}
\end{enumerate}
\begin{description}
\item[A new word] ...and its definition
        \begin{itemize}
        \item   Plus some more dot points.
        \end{itemize}
\end{description}
```

1. Hello!
2. World.
3. Another item
   - Second level
   - Second level item 2
4. (a) Nested levels
   - More levels!!!!
   - Even more dot points

   (b) Nested level item 2

A new word ...and its definition

- Plus some more dot points.

Some common environments

- `enumerate` – numbered list
- `itemize` – itemized list
- `description` – emphasises a particular word/phrase, with explanations following
- `flushleft` – left aligned with no paragraph indentation
- `center` – centre aligned (note US spelling!)
- `flushright` – right aligned
- `multicols` – multicolumn[1]

[1] Requires `multicol` package

3.3 Inline math vs Displayed math

3.3.1 Inline math

Math within a line is known as *inline math*.

Example 8

Listing 3.5 – Math

```
This is a quadratic equation:
$2x^2 + 5x - 2 = 0$
```

This is a quadratic equation: $2x^2 + 5x - 2 = 0$

Example 9

Listing 3.6 – Math

```
The quadratic formula is $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
```

The quadratic formula is $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Example 10

Listing 3.7 – Exercises

```
Simplify
\begin{multicols}{3}
\begin{enumerate}
\item   $3a^4 \times 2a$
\item   $5a^3 \times 3a^2 b^2$
\item   $(-3x) \times (-4x)$
\item   $\left(3a^2\right)^3$
\item   $\left(-2a^2 b\right)^5$
\item   $16x^8 \div 2x^2$
\item   $\dfrac{6x^3 y^2 z}{9x^2 y^4 z}$
\item   $\left(pq^3\right)^2 \times p^3 q \div \left(pq\right)^3$
\item   $x(2x + 1)^3 \times x^4 (2x + 1)$
\item   $t \times \dfrac{1}{t}$
\item   $\sqrt{x} \times \sqrt{x}$
\item   $\text{reciprocal of } a + b$
\end{enumerate}
\end{multicols}
```

Simplify

1. $3a^4 \times 2a$
2. $5a^3 \times 3a^2b^2$
3. $(-3x) \times (-4x)$
4. $(3a^2)^3$
5. $(-2a^2b)^5$
6. $16x^8 \div 2x^2$
7. $\dfrac{6x^3y^2z}{9x^2y^4z}$
8. $(pq^3)^2 \times p^3q \div (pq)^3$
9. $x(2x+1)^3 \times x^4(2x+1)$
10. $t \times \dfrac{1}{t}$
11. $\sqrt{x} \times \sqrt{x}$
12. reciprocal of $a + b$

Some miscellaneous math mode commands:

- `\underline{...}` – underline.
- `\overline{...}` – overline.
- `\underbrace{...}_{...}` – braces underneath the block of text.
- `\overbrace{...}_{...}` – braces over the block of text.
- `\underset{...}{...}` – sets the text underneath the block of text
- `\overset{...}{...}` – sets the text over the block of text
- `\text{...}` – allows normal text input inside math mode.
- `\intertext{...}` – inserts one line of text inside a multiple line displayed equation (See Section 3.3.2 on the following page)
- `\mathbb{R}` – $\mathbb{R}$ (also can use N, Z, Q, C for different sets)

Additional spacing

| |	`\,`	thin space	| |	`\:`	medium space	| |	`\ `	interword space
‖	`\!`	negative thin space	| |	`\;`	thick space			

3.3.2 Displayed math environments

Notice in Example 9 the quadratic equation's fraction was "cramped up". Use *displayed math* to avoid these problems when presenting larger equations.

Some further environments inside displayed math to consider:

Some common environments

- `\[ ... \]` – unnumbered, single displayed equation
- `equation` – numbered, singled displayed equation
- `gather`, `gather*` – multiple lines. (Starred version is unnumbered)
- `align`, `align*` – multiple lines, aligned at a certain position.
  - Variant: `aligned`, to be used inside a `gather` or `gather*` environment
- `multline`, `multline*` – breaks one single equation into multiple lines
- `cases` – typesets cases (useful for piecewise defined functions)

With exception of the first two, the remaining environments require the `amsmath` package to be loaded. See Section 4.1 on page 23.

Example 11

Listing 3.8 – `cases` environment

```
1 Solve simultaneously:
2 $\begin{cases}
3     x^2 = 4ay                 & \text{(parabola)} \\
4     x + py = ap^3 + 2ap       & \text{(normal at $P$)}
5 \end{cases}$
```

Solve simultaneously: $\begin{cases} x^2 = 4ay & \text{(parabola)} \\ x + py = ap^3 + 2ap & \text{(normal at } P\text{)} \end{cases}$

Example 12

Listing 3.9 – Quadratic equation using displayed math

```
1 \begin{gather*}
2 x^2 - 4x - 5 = 0 \\
3 (x - 5)(x + 1) = 0 \\
4 \therefore x = 5, -1
5 \end{gather*}
```

$$x^2 - 4x - 5 = 0$$
$$(x - 5)(x + 1) = 0$$
$$\therefore x = 5, -1$$

Example 13

Listing 3.10 – Derivation of the quadratic formula

```
Derivation of the quadratic formula
\begin{gather*}
\underbrace{ax^2 + bx + c}_{\div a} = \underset{\div a}{0} \\
x^2 + \frac{b}{a}x + \underset{-\frac{c}{a}}{\frac{c}{a}}
    = \underset{-\frac{c}{a}}{0}
\intertext{Completing the square,}
x^2 + \frac{b}{a} x + \left(\frac{b}{2a}\right)^2
    = \left(\frac{b}{2a}\right)^2 - \frac{c}{a} \\
\begin{aligned}
\left(x + \frac{b}{2a}\right)^2
    &= \frac{b^2}{4a^2} - \frac{4ac}{4a^2} \\
    &= \frac{b^2 - 4ac}{(2a)^2}
\end{aligned}
\intertext{Take square roots of both sides,}
\begin{aligned}
x + \frac{b}{2a} &= \frac{\pm \sqrt{b^2 - 4ac}}{2a} \\
x &= -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} \\
  &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\end{aligned}
\end{gather*}
```

Derivation of the quadratic formula

$$\underbrace{ax^2 + bx + c}_{\div a} = \underset{\div a}{0}$$

$$x^2 + \frac{b}{a}x + \underset{-\frac{c}{a}}{\frac{c}{a}} = \underset{-\frac{c}{a}}{0}$$

Completing the square,

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$$

$$\begin{aligned}\left(x + \frac{b}{2a}\right)^2 &= \frac{b^2}{4a^2} - \frac{4ac}{4a^2} \\ &= \frac{b^2 - 4ac}{(2a)^2}\end{aligned}$$

Take square roots of both sides,

$$\begin{aligned}x + \frac{b}{2a} &= \frac{\pm\sqrt{b^2 - 4ac}}{2a} \\ x &= -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\end{aligned}$$

Example 14

Listing 3.11 – `multline` environment

```
\begin{multline*}
(1 + x)^{2n} = \binom{2n}{0} + \binom{2n}{1} x + \binom{2n}{2} x^2  + \cdots \\
    \cdots + \binom{2n}{n} x^{n} + \binom{2n}{n + 1} x^{n + 1} + \cdots \\
    \cdots + \binom{2n}{2n - 1} x^{2n - 1} + \binom{2n}{2n} x^{2n}
             = \sum_{k = 0}^{2n} \binom{2n}{k} x^k
\end{multline*}
```

$$(1+x)^{2n} = \binom{2n}{0} + \binom{2n}{1}x + \binom{2n}{2}x^2 + \cdots \\ \cdots + \binom{2n}{n}x^n + \binom{2n}{n+1}x^{n+1} + \cdots \\ \cdots + \binom{2n}{2n-1}x^{2n-1} + \binom{2n}{2n}x^{2n} = \sum_{k=0}^{2n}\binom{2n}{k}x^k$$

Further exercises

Typeset part of the solutions that were handwritten from an old examination.

3.4 Cross referencing

Cross referencing in LaTeX works across text and math mode to provide seamless ways to refer to material within the same document. The basic commands for referencing:

- `\label{...}`
- `\ref{...}`
- `\eqref{...}`
- `\footnote{...}`
- `\pageref{...}`
- `\vref{...}`[2]

Note: As almost anything could be a label name, try prefixing with a suitable abbreviation to classify what the labels refer to:

- `chap:` chapter
- `subsec:` subsection
- `tab:` table
- `sec:` section
- `fig:` figure
- `eq:` equation

Wherever a `\label{...}` is placed, it will "remember" the chapter/section/subsection/equation number of its location. When referencing,

- `\ref{...}` will produce the correct chapter/section/subsection or equation number.
- `\pageref{...}` will produce the page number of the referenced chapter/section/subsection or equation number.
- `\vref{...}` will produce the chapter/section/subsection or equation *and* page number.

Example 15

Listing 3.12 – Labelling and Referencing

```
\subsection{Variable sized operators/delimiters}

Some symbols are variable in size. Edmodo will use displayed (as opposed to inline)
versions of these symbols for ease of viewing (For an explanation of Displayed vs
Inline equations, see Section \vref{sec:inline-display}).

...

\section{Inline math vs Displayed math}
\label{sec:inline-display}

\subsection{Inline math}
Math within a line is known as \emph{inline math}.
```

2.1.5 Variable sized operators/delimiters

Some symbols are variable in size. Edmodo will use displayed (as opposed to inline) versions of these symbols for ease of viewing (For an explanation of Displayed vs Inline equations, see Section 3.3 on page 15).

⋮

3.3 Inline math vs Displayed math

3.3.1 Inline math

Math within a line is known as *inline math*.

[2] Requires `varioref` package.

Two other useful commands for labelling of equations whilst in displayed math mode:

- `\notag` - turns *off* numbering for that line
- `\tag{...}` - uses a symbol for a custom tag (look up list of symbols from *A comprehensive list of symbols for LaTeX, p.22-23*)

Example 16

Listing 3.13 – Extension 2 integration

```
\begin{gather}
I   = \int_0^{\frac{\pi}{2}} \frac{e^{\sin x}}{e^{\sin x} + e^{\cos }} \: dx
    \tag{$\bigstar$}
\intertext{By using the result from above,}
\begin{aligned}
\therefore I
    &= \int_0^{\frac{\pi}{2}}
        \frac{e^{\sin \left(\frac{\pi}{2} - x\right)}}
            {e^{\sin \left(\frac{\pi}{2} - x\right)} + e^{\cos \left(\frac{\pi}{2} - x\right)}} \: dx
    &= \int_0^{\frac{\pi}{2}} \frac{e^{\cos x}}{e^{\cos x} + e^{\sin x}} \: dx
\end{aligned} \tag{$\blacktriangledown$}
\intertext{Adding $(\blacktriangledown)$ and $(\bigstar)$,}
\begin{aligned}
2I  &= \int_0^{\frac{\pi}{2}} \frac{e^{\cos x}}{e^{\cos x} + e^{\sin x}} \: dx
        + \int_0^{\frac{\pi}{2}} \frac{e^{\sin x}}{e^{\cos x} + e^{\sin x}} \: dx
    = \int_0^{\frac{\pi}{2}} \frac{e^{\sin x}
        + e^{\cos x}}{e^{\sin x} + e^{\cos x}}\: dx \\
    &= \int_0^{\frac{\pi}{2}} 1 \: dx = \frac{\pi}{2}
\end{aligned} \notag \\
\therefore I = \frac{\pi}{4} \label{eqn:answer}
\end{gather}
And the answer is shown in Equation \eqref{eqn:answer}.
```

$$I = \int_0^{\frac{\pi}{2}} \frac{e^{\sin x}}{e^{\sin x} + e^{\cos}}\, dx \tag{$\bigstar$}$$

By using the result from above,

$$\therefore I = \int_0^{\frac{\pi}{2}} \frac{e^{\sin\left(\frac{\pi}{2} - x\right)}}{e^{\sin\left(\frac{\pi}{2} - x\right)} + e^{\cos\left(\frac{\pi}{2} - x\right)}}\, dx \quad = \int_0^{\frac{\pi}{2}} \frac{e^{\cos x}}{e^{\cos x} + e^{\sin x}}\, dx \tag{$\blacktriangledown$}$$

Adding $(\blacktriangledown)$ and $(\bigstar)$,

$$\begin{aligned} 2I &= \int_0^{\frac{\pi}{2}} \frac{e^{\cos x}}{e^{\cos x} + e^{\sin x}}\, dx + \int_0^{\frac{\pi}{2}} \frac{e^{\sin x}}{e^{\cos x} + e^{\sin x}}\, dx = \int_0^{\frac{\pi}{2}} \frac{e^{\sin x} + e^{\cos x}}{e^{\sin x} + e^{\cos x}}\, dx \\ &= \int_0^{\frac{\pi}{2}} 1\, dx = \frac{\pi}{2} \end{aligned}$$

$$\therefore I = \frac{\pi}{4} \tag{12.1}$$

And the answer is shown in Equation (12.1).

Section 4

Packages to alter the look and feel

LaTeX It is easily extended via *packages* to enable additional symbols, layout structures and even drawing diagrams.

4.1 Package: $\mathcal{AMS}$ math

Use $\mathcal{AMS}$ is the American Mathematical Society, is the US equivalent of the Australian Mathematics Trust (AMT). AMS has a large presence at the tertiary level, and has written a large package to support additional command/environments/symbols.

Loading Insert the following into the preamble:

Listing 4.1 – Insert into preamble to load package

```
1 \usepackage{amsmath}
```

Commands/environments provided: See Section 3.3 on page 16 for a list of frequently used commands/environments.

Documentation On CTAN.

Example 17

Listing 4.2 – Using display style

```
1 The fundamental theorem of calculus is
2 $f(x) = \displaystyle \frac{d}{dx} \int^x f(t) \: dt$.
3
4 Alternatively, $\dfrac{d}{dx}$ also presents a displayed fraction
5     inside an inline math.
```

The fundamental theorem of calculus is $f(x) = \displaystyle \frac{d}{dx} \int^x f(t) \: dt$.

Alternatively, $\dfrac{d}{dx}$ also presents a displayed fraction inside an inline math.

4.2 Package: cancel

Use Allows cancellations in mathematics formulae to be shown.

Loading Insert the following into the preamble:

Listing 4.3 – Insert into preamble to load package

```
\usepackage[thicklines,Smaller]{cancel}
```

Commands provided:

- `\cancel{...}`
- `\cancelto{...}{...}`

Documentation On CTAN.

Example 18

Listing 4.4 – Using the cancel package

```
\[
\frac{\left(x^2 - 1\right)(x + 2)}{x^2 + 3x + 2}
    = \frac{(x - 1)\cancelto{1}{(x + 1)}\cancelto{1}{(x + 2)}}
        {\cancel{(x + 1)}\cancel{(x + 2)}} \\
    = (x - 1)
\]
```

$$\frac{(x^2-1)(x+2)}{x^2+3x+2} = \frac{(x-1)\cancelto{1}{(x+1)}\cancelto{1}{(x+2)}}{\cancel{(x+1)}\cancel{(x+2)}} = (x-1)$$

4.3 Package: esvect

Use Provides extensible arrows at the top of some characters for vector geometry notation.

Loading Insert the following into the preamble:

Listing 4.5 – Insert into preamble to load package

```
\usepackage{esvect}
```

Commands provided:

- `\vv{...}`

Documentation On CTAN.

Example

Example 19

Listing 4.6 – Using the esvect package

```
Hence show that $\vv{PQ} = \vv{SR}$.
```

Hence show that $\overrightarrow{PQ} = \overrightarrow{SR}$.

4.4 Package: geometry

Use Alters page layout (reduce margins etc)

Loading Insert the following into the preamble:

Listing 4.7 – Insert into preamble to load package

```
1 \usepackage[dvips,twoside,includeheadfoot,bindingoffset=1.5cm,
2     left=1.5cm,right=1.5cm,height=0.9\paperheight]{geometry}
```

Commands provided: See documentation. Package works in preamble declarations.

Documentation On CTAN.

Example Package works in the preamble declarations. No further example provided.

4.5 Package: hyperref

Use Allows hyperlinks in PDF that is generated, either manually through `\href{...}{...}` or other cross references (`\ref{...}`, `\pageref{...}`, `\eqref{...}`, `\vref{...}`). (See also `url` at Section 4.11 on page 29)

Loading Insert the following into the preamble:

Listing 4.8 – Insert into preamble to load package

```
1 \usepackage{hyperref}
```

Commands/environments provided:

- `\href{...}{...}`

Documentation On CTAN.

Example 20

Listing 4.9 – Using `hyperref`

```
1 The \href{http://www.smh.com.au}{SMH} is sometimes
2     also known as the ``The Sydney Middle-class Herald''.
```

The SMH is sometimes also known as the "The Sydney Middle-class Herald".

4.6 Package: multicol

Use Allows typesetting in multiple columns

Loading Insert the following into the preamble:

Listing 4.10 – Insert into preamble to load package

```
1 \usepackage{multicol}
```

Commands/environments provided:

- `\begin{multicols}{..} ... \end{multicols}`
- `\columnbreak`

Documentation On CTAN.

Example See Example 10 on page 16.

4.7 Package: lhelp

Use Provides additional supporting commands

Loading Insert the following into the preamble:

Listing 4.11 – Insert into preamble to load package

```
1 \usepackage[units]{lhelp}
```

Commands/environments provided:

- \degree
- \celcius
- \fahren

Documentation On CTAN.

Example 21

Listing 4.12 – Using lhelp

```
1 There $2\pi$ radians for every 360\degree.
```

There 2π radians for every 360°.

4.8 Package: numprint

Use Typesets numbers, automatically inserting thousands separators, exponents and units

Loading Insert the following into the preamble:

Listing 4.13 – Insert into preamble to load package

```
1 \usepackage[np,autolanguage]{numprint}
2
3 \npthousandsep{\hspace{0.2em}}
```

Commands/environments provided:

- \np[...]{...}

Documentation On CTAN.

Example 22

Listing 4.14 – Using the numprint

```
1 One billion can be represented as: \np{1000000000} or \np{1e9}.
2 One billion kilometres inside math mode: $\np[km]{1e9}$.
```

One billion can be represented as: 1 000 000 000 or 1×10^9. One billion kilometres inside math mode: 1×10^9 km.

4.9 Package: paralist

Use Typesets enumerated and itemized lists horizontally. Useful for typesetting answers to exercises (to save space)

Loading Insert the following into the preamble:

Listing 4.15 – Insert into preamble to load package

```
\usepackage{paralist}
```

Commands/environments provided:

- `\begin{inparaenum}[...] ... \end{inparaenum}`
- `\begin{inparaitem}[...] ... \end{inparaitem}`

Documentation On CTAN.

Example 23

Listing 4.16 – Using `inparaenum` with optional argument (inside square brackets) from `paralist`

```
\scriptsize \paragraph{Answers:}
\begin{inparaenum}[\bfseries 1.]
\item   $\frac{1}{2}\log_e \left(x^2 - 1\right) + C$
\item   $\frac{1}{2}\log_e \left(2x + 3\right) + C$
\item   $\frac{1}{\log_e 2}e^{x \log_e 2} + C$
\end{inparaenum}
```

Answers: **1.** $\frac{1}{2}\log_e\left(x^2-1\right)+C$ **2.** $\frac{1}{2}\log_e\left(2x+3\right)+C$ **3.** $\frac{1}{\log_e 2}e^{x\log_e 2}+C$

4.10 Package: stmaryrd

Use Typesets additional symbols, in particular, the ||| symbol used for similarity.

Loading Insert the following into the preamble:

Listing 4.17 – Insert into preamble to load package

```
\usepackage{stmaryrd}
```

Commands/environments provided:

- `\interleave`

Documentation On CTAN.

Example 24

Listing 4.18 – Using `stmaryrd`

```
$\therefore \triangle ABC \interleave \triangle XYZ$ (equiangular)
```

$\therefore \triangle ABC \interleave \triangle XYZ$ (equiangular)

4.11 Package: `url`

Use Allows typesetting URLs. Complements `hyperref` (See Section 4.5 on page 26)

Loading Insert the following into the preamble:

Listing 4.19 – Insert into preamble to load package

```
1 \usepackage{url}
```

Commands/environments provided:

- `\url{...}`

Documentation On CTAN.

Example 25

Listing 4.20 – Using `url`

```
1 \url{www.smh.com.au} is the URL for the ``The Sydney Middle-class Herald''.
```

www.smh.com.au is the URL for the "The Sydney Middle-class Herald".

4.12 Package: `xlop`

Use Allows typesetting vertical addition/subtraction/multiplication. Most useful for documents involving Year 7 math.

Loading Insert the following into the preamble:

Listing 4.21 – Insert into preamble to load package

```
1 \usepackage{xlop}
2
3 \opset{resultstyle.d=\normalsize,intermediarystyle=\normalsize, %
4     resultstyle=\normalsize,voperator=bottom,carrystyle=\tiny}
```

Commands/environments provided:

- `\opadd{...}{...}`
- `\opsub{...}{...}`
- `\opmul{...}{...}`

Documentation On CTAN.

Example 26

Listing 4.22 – Using the `xlop` package

```
1 \opadd{155}{296} \quad \opsub{5325}{4853}
```

$$\begin{array}{r} {\scriptstyle 1\ \ 1\ \ \ \ } \\ 1\,5\,5 \\ +\,2\,9\,6 \\ \hline 4\,5\,1 \end{array} \qquad \begin{array}{r} \\ 5\,3\,2\,5 \\ -\,4\,8\,5\,3 \\ \hline 4\,7\,2 \end{array}$$

4.13 Package: varioref

Use References chapter/section/subsection/equation numbers with page numbers.

Loading Insert the following into the preamble:

Listing 4.23 – Insert into preamble to load package

```
1 \usepackage{varioref}
```

Commands/environments provided:

- `\vref{...}`

Documentation On CTAN.

Example See Example 15 on page 21 (this reference was created of course, using `\vref{...}`)

Section 5

Via WinEdt

5.1 Compilation

WinEdt is the editor that assists in the composition and compilation of LaTeX code. Previously, ShareLaTeX took care of the editing, compilation and output display, however it is not available for use offline.

5.2 Useful shortcut keys in WinEdt

- `Ctrl-Shift-X` – TeXify. Compiles LaTeX code to dvi.
- `Ctrl-Shift-P` – PDFTeXify. Compiles LaTeX code to PDF.
- `Ctrl-=` – Show line numbers in WinEdt
- `Shift-Esc` – cancel compilation (gets out of errors!)
- `\begin{...}}` – note double closing braces – automatically inserts `\end{...}` to close off environment (See Section 3.2.3)
- `\end{{` –automatically closes previous `\begin{...}` environment.

Part II

Advanced techniques

Section 6

Custom commands to reduce typing

Whilst LaTeX is terrific with 99% of its mathematical input, there are some peculiarities with the NSW syllabuses that require some modification. The entire file containing the renewed commands can be downloaded from the NSWSSLUG Edmodo group `https://edmo.do/j/scwm3v`.

6.1 Topics: Basic Arithmetic, Complex Numbers, Inverse Functions

LaTeX was initially geared towards mathematics faculties in the United States. Some symbols/log-like operators differ from that of NSW in some minor ways. The table below illustrates the differences:

Symbol/Log-like operator	United States	NSW
Co-secant	$\csc x$ (`\csc x`)	$\operatorname{cosec} x$ (`\operatorname{cosec} x`)
Inverse sine (arcsine)	$\arcsin x$ (`\arcsin x`)	$\sin^{-1} x$ (`\sin^{-1} x`)
Inverse cosine (arccosine)	$\arccos x$ (`\arccos x`)	$\cos^{-1} x$ (`\cos^{-1} x`)
Inverse tangent (arctan)	$\arctan x$ (`\arctan x`)	$\tan^{-1} x$ (`\tan^{-1} x`)
Permutation	$P(n,r)$ (`P(n,r)`)	${}^{n}P_{r}$ (`^n P_r`)
Combination	$C(n,r)$ (`C(n,r)`)	${}^{n}C_{r}$ (`^n C_r`)

Table 6.1 – Differences in mathmatical typesetting/terminology

To save time and effort, some new commands have been defined to speed up inserting these log-like operators.

Loading Insert the following into the preamble:

Listing 6.1 – Insert into preamble to load package

```
\usepackage{pstricks} % for \dotX{} and \ddotX{}
\input{environments.tex}
```

Commands/environments provided:

- `\abs{...}`
- `\cosec`
- `\ASIN`
- `\ACOS`
- `\ATAN`
- `\Re`
- `\Im`
- `\Arg`
- `\nPr{..}{..}`
- `\nCr{..}{..}`
- `\dotX{..}`
- `\ddotX{..}`

Inside `environments.tex`, the following code exists:

Listing 6.2 – `environments.tex` code snippets

```
\newcommand{\cosec}{\operatorname{cosec}}
\newcommand{\nPr}[2]{\ensuremath{^{#1}P_{#2}}}
\newcommand{\nCr}[2]{\ensuremath{^{#1}C_{#2}}}
\newcommand{\dotX}[1]{\ensuremath{\overset{\psdot[dotscale=0.66]}{#1}}}
\newcommand{\ddotX}[1]{\ensuremath{\overset{\psdot[dotscale=0.66] %
    \hspace{0.55ex}\psdot[dotscale=0.66]}{#1}}}
\newcommand{\ATAN}{\ensuremath{\tan^{-1}}}
\newcommand{\ACOS}{\ensuremath{\cos^{-1}}}
\newcommand{\ASIN}{\ensuremath{\sin^{-1}}}
\renewcommand{\Re}{\operatorname{Re}}
\renewcommand{\Im}{\operatorname{Im}}
\newcommand{\Arg}{\ensuremath{\operatorname{Arg}}}
\newcommand{\abs}[1]{\ensuremath{\left| #1 \right|}}
```

Example 27

Listing 6.3 – Custom math symbols

```
\begin{itemize}
\item   Basic arithmetic:
        \[ \abs{\int_a^b \dfrac{3x + 1}{4x + 5} \: dx} \]
\item   Trigonometric:
        \[ \cosec x \quad \ASIN x \quad \ACOS x \quad \ATAN x \]
\item   Complex numbers:
        \[ \Re\left(z^2 + 2z\right) \quad  \Im\left(z^2 + 2z\right)
                \quad \Arg \left(z^2 + 2z\right) \]
\item   Permutation/Combination:
        \[ \nPr{5}{3} \quad \nCr{n + 2}{k - 2} \]
\item   Applications of calculus to the physical world:
        \[ \dotX{x} = -n^2 \left(a^2 - x^2\right) \qquad \ddotX{x} = -n^2 x \]
\end{itemize}
```

- Basic arithmetic: (absolute value automatically "grows" with the height of the included text)

$$\left| \int_a^b \frac{3x+1}{4x+5}\, dx \right|$$

- Trigonometric:

$$\operatorname{cosec} x \quad \sin^{-1} x \quad \cos^{-1} x \quad \tan^{-1} x$$

- Complex numbers:

$$\operatorname{Re}\left(z^2+2z\right) \quad \operatorname{Im}\left(z^2+2z\right) \quad \operatorname{Arg}\left(z^2+2z\right)$$

- Permutation/Combination:

$${}^{5}P_{3} \quad {}^{n+2}C_{k-2}$$

- Applications of calculus to the physical world: (more obvious dots above the x)

$$\dot{x} = -n^2\left(a^2 - x^2\right) \qquad \ddot{x} = -n^2 x$$

6.2 Vectors

Commands/environments provided:

- `\cvv{...}{...}`
- `\vb{...}`
- `\vijk{...}{...}{...}`
- `\cvvv{...}{...}{...}`
- `\vij{...}{...}`
- `\proj`

Example 28

Listing 6.4 – Custom math symbols

```
\begin{itemize}
\item   Find the angle between $\vb{a} = \cvv{3}{-2}$ and $\vb{b} = \cvv{1}{7}$
\item   Prove that $\vb{a} = \vij{\cos \alpha}{\sin \alpha}$
\item   If $\vb{p} = \vij{3}{-5}$ and $\vb{q} = \vij{-1}{-2}$, find $\abs{\vb{p} - 2\vb{q}}$
\item   $\vb{\dotX{r}} = \lambda \cvvv{3}{t}{t^2}$
\end{itemize}
```

- Find the angle between $\underset{\sim}{\mathrm{a}} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ and $\underset{\sim}{\mathrm{b}} = \begin{pmatrix} 1 \\ 7 \end{pmatrix}$
- Prove that $\underset{\sim}{\mathrm{a}} = \cos\alpha\,\underset{\sim}{\mathrm{i}} + \sin\alpha\,\underset{\sim}{\mathrm{j}}$
- If $\underset{\sim}{\mathrm{p}} = 3\underset{\sim}{\mathrm{i}} - 5\underset{\sim}{\mathrm{j}}$ and $\underset{\sim}{\mathrm{q}} = -\underset{\sim}{\mathrm{i}} - 2\underset{\sim}{\mathrm{j}}$, find $\left|\underset{\sim}{\mathrm{p}} - 2\underset{\sim}{\mathrm{q}}\right|$
- $\dot{\underset{\sim}{\mathrm{r}}} = \lambda \begin{pmatrix} 3 \\ t \\ t^2 \end{pmatrix}$

Note Simply renew the command `\vb{...}{...}` if either undertilde/math roman or math boldface isn't preferred for vector variables:

1. Load the `accents` package
2. Renew the command to `\undertilde{\mathrm{#1}}` or `\mathbf{#1}`.

See Listing 6.6 on the next page.

Example 29

Listing 6.5 – Renewing vector variables - insert into preamble

```
\renewcommand{\vb}[1]{\ensuremath{\undertilde{#1}}}
\begin{itemize}
\item   Find the angle between $\vb{a} = \cvvv{3}{-2}{1}$ and $\vb{b} = \cvvv{1}{7}{8}$
\item   Prove that $\vb{a} = \vij{\cos \alpha}{\sin \alpha}$
\item   If $\vb{p} = \vijk{3}{-5}{t}$ and
        $\vb{q} = \vijk{-1}{-2}{t^2}$, find $\abs{\vb{p} - 2\vb{q}}$
\end{itemize}
A vector representation of this force could be:
\begin{multicols}{3}
\begin{enumerate}[label=(\Alph*)]
\item   $\vij{9\sqrt{3}}{9}$
\item   $\vij{-9}{9\sqrt{3}}$
\item   $\vij{-9\sqrt{3}}{9}$
\item   $-9\, \vb{i} -9\sqrt{3}\, \vb{j}$
\item   $\vij{9\sqrt{3}}{-9}$
\end{enumerate}
\end{multicols}
\end{boxexample}
```

- Find the angle between $\underset{\sim}{a} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$ and $\underset{\sim}{b} = \begin{pmatrix} 1 \\ 7 \\ 8 \end{pmatrix}$
- Prove that $\underset{\sim}{a} = \cos\alpha\,\underset{\sim}{i} + \sin\alpha\,\underset{\sim}{j}$
- If $\underset{\sim}{p} = 3\underset{\sim}{i} - 5\underset{\sim}{j} + t\,\underset{\sim}{k}$ and $\underset{\sim}{q} = -\underset{\sim}{i} - 2\underset{\sim}{j} + t^2\,\underset{\sim}{k}$, find $\left|\underset{\sim}{p} - 2\underset{\sim}{q}\right|$

A vector representation of this force could be:

(A) $9\sqrt{3}\underset{\sim}{i} + 9\underset{\sim}{j}$ (C) $-9\sqrt{3}\underset{\sim}{i} + 9\underset{\sim}{j}$ (E) $9\sqrt{3}\underset{\sim}{i} - 9\underset{\sim}{j}$

(B) $-9\underset{\sim}{i} + 9\sqrt{3}\underset{\sim}{j}$ (D) $-9\underset{\sim}{i} - 9\sqrt{3}\underset{\sim}{j}$

Important note

(!) `\vij{...}{...}` and `\vijk{...}{...}{...}` unfortunately does not work with `\dotX{...}` or `\ddotX{...}` inside the arguments.

Workaround: simply use `\dot{...}` or `\ddot{...}`. It produces a smaller dot, but gets the job done.

Inside `environments.tex`, the following code exists:

Listing 6.6 – `environments.tex` code snippets

```
\newcommand{\cvv}[2]{\ensuremath{\begin{pmatrix} #1 \\ #2 \end{pmatrix}}}
\newcommand{\cvvv}[3]{\ensuremath{\begin{pmatrix} #1 \\ #2 \\ #3 \end{pmatrix}}}
\newcommand{\vb}[1]{\ensuremath{\mathbf{#1}}}
\newcommand{\vij}[2]{\ensuremath{ % remove the '1' if it's '-1'
    \ifthenelse{\equal{#1}{0}}{}{
        \ifthenelse{\equal{#1}{-1}}{-}{
            \ifthenelse{\equal{#1}{1}}{}{#1\,} %
        } \mathbf{i} % leave a blank if it's '1'
    }
    \ifthenelse{\equal{#2}{0}}{}{
        \ifthenelse{\equal{#2}{-1}}{-}{ % remove the '1' if it's '-1'
            \ifthenelse{\equal{#2}{1}}{+}{ % leave a blank if it's '1'
                \noexpandarg\IfBeginWith{#2}{-}{#2 \,}{+ #2 \,} % check if it's negative or not
            }
        } \mathbf{j}
    }
}}
\newcommand{\vijk}[3]{\ensuremath{ % remove the '1' if it's '-1'
    \ifthenelse{\equal{#1}{0}}{}{
        \ifthenelse{\equal{#1}{-1}}{-}{
            \ifthenelse{\equal{#1}{1}}{}{#1\,} %
        } \mathbf{i} % leave a blank if it's '1'
    }
    \ifthenelse{\equal{#2}{0}}{}{
        \ifthenelse{\equal{#2}{-1}}{-}{ % remove the '1' if it's '-1'
            \ifthenelse{\equal{#2}{1}}{+}{ % leave a blank if it's '1'
                \noexpandarg\IfBeginWith{#2}{-}{#2 \,}{+ #2 \,} % check if it's negative or not
            }
        } \mathbf{j}
    }
    \ifthenelse{\equal{#3}{0}}{}{
        \ifthenelse{\equal{#3}{-1}}{-}{ % remove the '1' if it's '-1'
            \ifthenelse{\equal{#3}{1}}{+}{ % leave a blank if it's '1'
                \noexpandarg\IfBeginWith{#3}{-}{#3 \,}{+ #3 \,} % check if it's negative or not
            }
        } \mathbf{k}
    }
}}
\newcommand{\proj}{\ensuremath{\operatorname{proj}}}
```

6.3 Syllabus/learning symbols

Loading Insert the following into the preamble:

Listing 6.7 – Insert into preamble to load package

```
\input{environments.tex}
```

Commands/environments provided:

- \Adv
- \ExtOne
- \ExtTwo
- \RefSheet
- \URL
- \Review
- \Enrichment
- \Extension
- \Warn
- \Literacy
- \Memorise
- \Understand

Example 30

Listing 6.8 – Explanation of Symbols

```
\begin{tabular}{lp{13cm}}
\textbf{Symbol} & \textbf{Explanation}
 \\[1ex]
\TwoU       & Mathematics content. Inserted into Extension 1/2 programs
                to meet prerequisites \\[2ex]
\ExtOne     & Mathematics Extension 1 exclusive content. \\[2ex]
\ExtTwo     & Mathematics Extension 2 exclusive content. \\[2ex]
\Literacy   & Content requiring literacy skills and additional teacher guidance. \\[2ex]
\Extension  & Extension material that may involve acceleration. \\[2ex]
\Enrich     & Enrichment material. See Resource list for references. \\[2ex]
\Review     & Revision content. Where possible, references to the
                first instance of the content being taught will be provided. \\[2ex]
\Warn       & Alert! Heed additional comment provided!
                Also, content may not be in usual prescribed textbook. \\[2ex]
\end{tabular}
```

Symbol	Explanation
(A)	Mathematics Advanced content. Inserted into Extension 1/2 programs to meet prerequisites
(X1)	Mathematics Extension 1 exclusive content.
(X2)	Mathematics Extension 2 exclusive content.
(L)	Content requiring literacy skills and additional teacher guidance.
(E)	Extension material that may involve acceleration.
(N)	Enrichment material. See Resource list for references.
(R)	Revision content. Where possible, references to the first instance of the content being taught will be provided.
(!)	Alert! Heed additional comment provided! Also, content may not be in usual prescribed textbook.

6.4 Exam/worksheet related commands

Loading Insert the following into the preamble:

Listing 6.9 – Insert into preamble to load custom commands

```
\usepackage{calc, enumitem}
\usepackage[normalem]{ulem}
\input{environments.tex}
```

Commands/environments provided:

- Cloze:
  - `\Cloze{...}`
  - `\ShowClozetrue`
  - `\ShowClozefalse`
- `\Lines{...}`
- `\marks{..}`
- `\begin{question}{..} ... \end{question}`
- `\begin{answers} ... \end{answers}`

Example 31

`\Cloze{...}`: Basically provides a blank space ("cloze") where student can fill out with a word. Line height and cloze width is automatically calculated based on the length of the text inserted.

Requires the `ulem` package with `normalem` option.

Listing 6.10 – Cloze custom command

```
\usepackage[normalem]{ulem}
...
\begin{document}
...
\ShowClozetrue

An \Cloze{even} function has symmetry about the $y$ axis. \\

\ShowClozefalse

Use the \Cloze{chain} \Cloze{rule} when differentiating! \\

\ShowClozetrue

It is possible to insert math:
\Cloze{$\displaystyle \int_a^b f(x) \: dx$} and yet not show it later:
\ShowClozefalse \Cloze{$z = r\left(\cos \theta + i\sin \theta\right)$}
...
\end{document}
```

An ..... even ..... function has symmetry about the y axis.

Use the .... when differentiating!

It is possible to insert math: ..... $\int_a^b f(x)\,dx$ ..... and yet not show it later: .....

Example 32

- `\Lines{...}`: Provides blank lines to write on. Super useful for junior exams and/or "extended response".
- `\begin{answers}... \end{answers}` allows for slightly more answers, e.g. an entire block of questions instead of just a continuous single line of answers.

Listing 6.11 – Lines custom command

```
\begin{multicols}{2}
\begin{enumerate}[label=(\alph*)]
\item   \ExtOne Evaluate $\displaystyle \int \sin^2 x \: dx$

        \Lines{2}
\item   \Warn Evaluate $\displaystyle \int \tan^2 x \: dx$

        \Lines{1}
\end{enumerate}
\end{multicols}
\begin{answers}
\begin{inparaenum}[(a)]
\item   $\frac{1}{2}x - \frac{1}{4} \sin 2x + C$
\item   $\tan x - x + C$
\end{inparaenum}
\end{answers}
```

(a) (x1) Evaluate $\int \sin^2 x \, dx$

.....

.....

(b) (!) Evaluate $\int \tan^2 x \, dx$

.....

Answers

(a) $\frac{1}{2}x - \frac{1}{4}\sin 2x + C$ (b) $\tan x - x + C$

Example 33

`\marks{...}`: Provides a macro to insert the number of marks in a set of solutions. Automatically adjusts for 1 mark (removes 's').

Listing 6.12 – Marks custom command

```
\begin{enumerate}
\item   \marks{3}
        \[ x = 2 \]
\item   \marks{1}
        \begin{gather*}
        \dotX{x} = \cos (2t + \phi) \\
        \therefore x = \frac{1}{2} \sin (2t + \phi)
        \end{gather*}
\end{enumerate}
```

1. (3 marks)

$$x = 2$$

2. (1 mark)

$$\dot{x} = \cos(2t + \phi)$$
$$\therefore x = \frac{1}{2}\sin(2t + \phi)$$

Example 34

`\begin{question}{..} ... \end{question}`: A saviour to anyone trying to write an exam and put the $#%& marks on the right of the page, bolded, and aligned to the first line of the question. **Do NOT load the `exam` package!**

Listing 6.13 – Question environment

```
\setenumerate[1]{label={\textbf{Question \arabic*}}, ref=\arabic*, %
    labelsep=1em,labelindent=0pt, labelsep=0ex, leftmargin=*, %
    listparindent=0pt,widest={10}, itemsep=2ex,align=left}
\setenumerate[2]{itemsep=1.5ex,align=left,label=(\alph*), %
    itemsep=2.5ex,labelsep=1em,listparindent=0pt,leftmargin=-3em}
\setenumerate[3]{labelsep=1em,itemsep=1.5ex,leftmargin=*}

\begin{enumerate}
\item   \
    \begin{enumerate}
    \item   \begin{question}{2}
            Evaluate:
            \[ \int_1^{a^2} \log_e \left(x^2 + 2x + 1\right) \: dx \]
            \end{question}
    \item   \begin{question}{1}
            Hence or otherwise, do something else.
            This line obviously has too much on it but the environment will
            automatically put the stuff on the next line
            without overwriting into the marks!
            \end{question}
    \end{enumerate}
\end{enumerate}
```

Question 1

(a) Evaluate: **2**

$$\int_1^{a^2} \log_e \left(x^2 + 2x + 1\right)\, dx$$

(b) Hence or otherwise, do something else. This line obviously has too much on it but the environment will automatically put the stuff on the next line without overwriting into the marks! **1**

Part III

Graphics

Section 7

Introduction to PSTricks

PSTricks stands for *PostScript Tricks*, a set of macros for the LaTeX markup language written by Professor Timothy Van Zandt in the the early 1990s. PSTricks gives users direct access to the PostScript language found in many high end printers. In turn, PostScript is a page description language that tells the printer what to draw on the page. Being based on vector graphics, it is very efficient at drawing lines, curves and text.

The usage of a printer's page description language to describe electronic documents. However it should not come as a surprise when one realises that Adobe invented PostScript and in fact PDF files are basically containers for PostScript information. Since it is possible to have LaTeX output to PostScript, this functionality will be exploited to produce exceptionally looking diagrams.

Assumed knowledge

- Basic LaTeX commands

Important note

The following must be in the preamble for graphics to look consistent

Example 35

Listing 7.1 – Setting some parameters straight

```
1 \psset{showorigin=false,arrowscale=1.55,linewidth=0.4pt,xAxisLabel=$x$,yAxisLabel=$y$,ury=0.5cm}
```

Section 8

Basics

8.1 Grids, coordinate axes, curve sketching

8.1.1 Grids

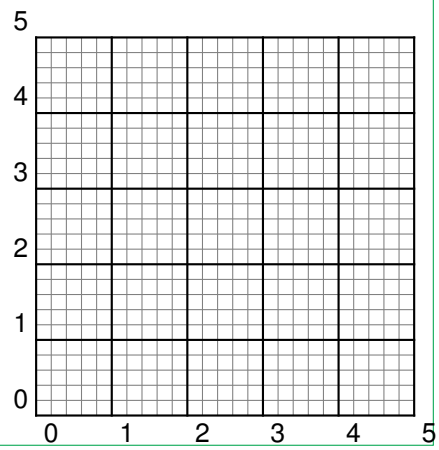
One of the most frustrating experiences is to draw a grid for student use. PSTricks has a macro for this.

Example 36

An elementary 5×5 grid

Listing 8.1 – A 5×5 grid

```
\begin{pspicture}(0,0)(5,5)
\psgrid
\end{pspicture}
```



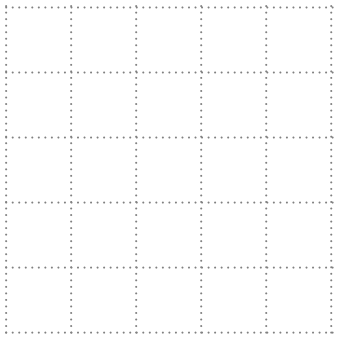
Most of the time a simpler grid is required without the subdivisions nor the automatic numbering and lighter gridlines with 10 dots per grid:

Example 37

A refined, elementary 5×5 grid with scale 1 unit = 0.86 cm

Listing 8.2 – A 5×5 grid

```
\psset{unit=0.86cm}
\begin{pspicture}(0,0)(5,5)
\psgrid[gridcolor=gray,
    griddots=10,gridlabels=0pt,subgriddiv=1]
\end{pspicture}
```



8.1.2 Dots, lines and basic shapes

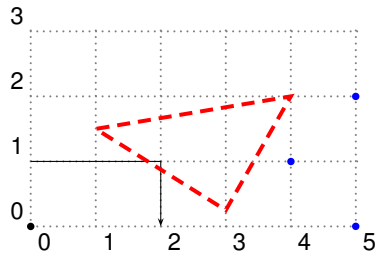
These elements are extremely easy to draw.

Example 38

Building on Example 37

Listing 8.3 – A 5×2 grid with dots, lines & basic shapes

```
\psset{unit=0.86cm}
\begin{pspicture}(0,0)(5,4)
\psgrid[griddots=10,gridcolor=gray,
    gridlabels=9pt,subgriddiv=1](0,0)(5,3)
\psdot(0,0)
\psdots[linecolor=blue](4,1)(5,0)(5,2)
\psline[arrows=->](0,1)(2,1)(2,0)
\pspolygon[linestyle=dashed,linecolor=red,
    linewidth=1.5pt](1,1.5)(4,2)(3,0.25)
\end{pspicture}
```



You may wonder why `\begin{pspicture}` has the upper right coordinates at $(5, 4)$ whilst the `\psgrid` was specified to only go up to $(5, 3)$. At times, some gaps may be required so that the diagram doesn't touch the text above. This example highlights the fact that `\psgrid` is not fixed to expand to the largest possible space given under the `pspicture` environment.

8.1.3 Coordinate axes

To draw coordinate axes, PSTricks has two macros,

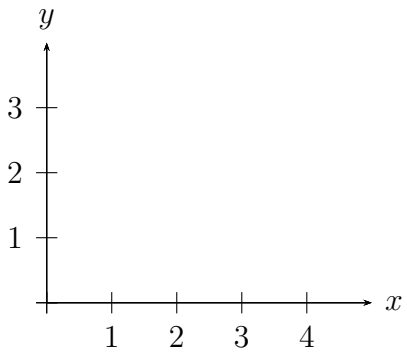
- `psaxes`
- `psgraph`

Example 39

Drawing coordinate axes – `psaxes`

Listing 8.4 – Coordinate axes

```
\psset{unit=0.86cm}
\begin{pspicture}(0,0)(5,4)
\psaxes{->}(0,0)(0,0)(5,4)[$x$,0][$y$,90]
\end{pspicture}
```



- The optional arguments for `\psaxes` specifies the placement angle for the x and y axis labels.

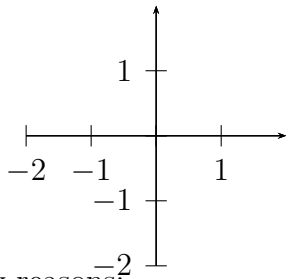
Most of the time however, the coordinate axes will need to show the negative x and y as well.

Example 40

Drawing coordinate axes with negative axes

Listing 8.5 – Coordinate axes

```
\psset{unit=0.86cm}
\begin{pspicture}(-2,-2)(2,2)
\psaxes{->}(0,0)(-2,-2)(2,2)
\end{pspicture}
```



Unfortunately this is not as pretty as it could be, for a few reasons:

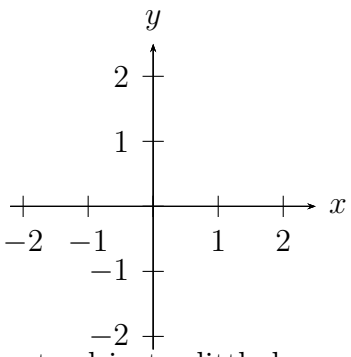
- The negative x and y axes end at -2 whilst the positive ends at 1
- The negative x and y ticks at -2 makes the axes look like it ends at -2.

Example 41

(via `pspicture` and `psaxes`) Drawing coordinate axes with negative axes, fixing up the ticks and final ordinates:

Listing 8.6 – Coordinate axes

```
\psset{unit=0.86cm}
\begin{pspicture}(-2.2,-2.2)(3,3)
\psaxes{->}(0,0)(-2.2,-2.2)(2.5,2.5)[$x$,0][$y$,90]
\end{pspicture}
```



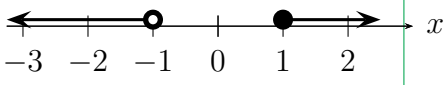
By tweaking the lower left corner of the coordinate axes to extend just a little beyond $(-2, -2)$ and extending the upper right corner to $(2.5, 2.5)$, the negative axes and labels no longer are truncated.

Example 42

(via `pspicture` and `psaxes`) Drawing a number line with the x axis only:

Listing 8.7 – Coordinate axes

```
\psset{unit=0.86cm}
\begin{pspicture}(-3.25,-1)(3.5,1.5)
\psaxes[yAxis=false]{->}(0,0)(-3.25,0)(3,0)[$x$,0][$$,0]
\psset{arrowscale=1.75}
\pstLineAB[offset=0.1,linewidth=1.15pt,arrows=*->]{A}{B}
\pstLineAB[offset=-0.1,linewidth=1.15pt,arrows=o->]{C}{D}
\end{pspicture}
```



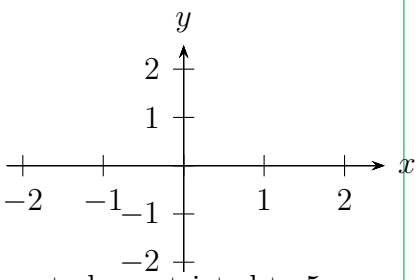
- The `yAxis=false` option will turn off the y axis
- However, if `[$x$,0]` is specified, then with the y axis turned off it also needs a 'blank' axis label, hence the additional `[$$,0]`.
- Lines in PSTricks have a direction, hence the offset needs to change for CD, as it runs in the negative direction.

Example 43

(via `psgraph`) Drawing coordinate axes with negative axes, fixing up the ticks and final ordinates:

Listing 8.8 – Coordinate axes

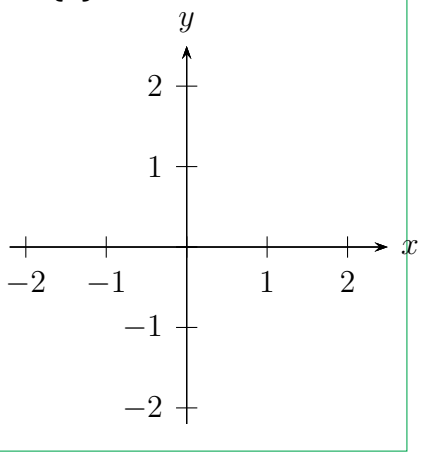
```
\begin{psgraph}{->}(0,0)(-2.2,-2.2)(2.5,2.5){5cm}{3cm}
\end{psgraph}
```



The *mandatory* arguments `{5cm}` and `{3cm}` allows the picture to be restricted to 5 cm horizontally, and 3 cm vertically. Good for documents where space may be limited, or when a graph needs to be fitted to certain constraints.
To make the vertical axis the same scale as the horizontal, use `{!}`:

Listing 8.9 – Coordinate axes

```
\begin{psgraph}{->}(0,0)(-2.2,-2.2)(2.5,2.5){5cm}{!}
\end{psgraph}
```



Section 9

Advanced graphics

9.1 Curve sketching

One of the best uses of PSTricks is to sketch basic curves. However, it is not possible to sketch more difficult curves without knowledge of *Reverse Polish Notation*.

9.1.1 Infix vs. Reverse Polish Notation (RPN)

When a mathematical expression is written, i.e. $\frac{1}{2}(3 \div 2 + 4)$, we are taught to the acronym "BODMAS" (**B**rackets **D**ivision **M**ultiplication **A**ddition **S**ubtraction) to understand the order of operations. That is,

$$\frac{1}{2}(3 \div 2 + 4) = \overset{\text{read from left to right} \rightarrow}{\Big((3 \div 2) + 4\Big)} \times \frac{1}{2} = \frac{11}{4} \tag{1.1}$$

PostScript understands nothing of the infix system. Rather, it uses RPN to perform arithmetic. In fact, the middle expression in (1.1) has already shown how to write $\frac{1}{2}(3 \div 2 + 4)$ in RPN.

RPN is otherwise known as a "last-in-first-out stack" for computers. Diagrammatically, a stack is a column of memory locations that can contain numbers and operations $+$, $-$, $\times$, $\div$, log, sin etc. The stack used under PSTricks will accept native PostScript commands.

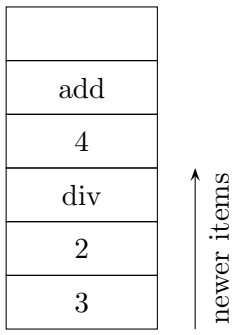


Figure 9.1 – Diagrammatic representation of a LIFO stack

So to input the expression in Equation 1.1 in RPN, simply type it out in the order that it should be processed from left to right, i.e.

$\frac{1}{2}(3 \div 2 + 4)$ `3 2 div 4 add 1 2 div mul`

Function	Example	Operation	Result
`add`	`6 4 add`	$6 + 4$	10
`sub`	`6 4 sub`	$6 - 4$	2
`mul`	`6 4 mul`	6×4	24
`div`	`6 4 div`	$6 \div 4$	1.5
`exp`	`4 3 exp`	4^3	64
`sqrt`	`2 sqrt`	$\sqrt{2}$	$1.41421356\cdots$
`abs`	`-4 abs`	$\lvert -4 \rvert$	4
`neg`	`5 neg`	$-(5)$	-5
`ln`	`7 ln`	$\log_e 7$	$1.945910\cdots$
`log`	`100 log`	$\log_{10} 100$	2
`def`	`/x 5 def \Func`	$f(5)$	Evaluate `\Func` with $x = 5$

Table 9.1 – PostScript commands useful for curve sketching

9.1.2 Basic curve sketching

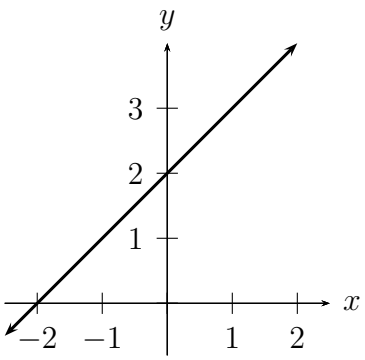
This section uses the the command `\psplot` extensively as well as the content in the earlier parts of this section.

Example 44

Sketching the graph $y = x + 2$,

Listing 9.1 – $y = x + 2$

```
\psset{unit=0.86cm}
\begin{pspicture}(-2.5,-1)(3,4)
\psaxes{->}(0,0)(-2.5,-0.8)(2.5,4)[$x$,0][$y$,90]
\psplot[linewidth=1.15pt,arrows=<->]
    {-2.5}{2}{x 2 add}
\end{pspicture}
```

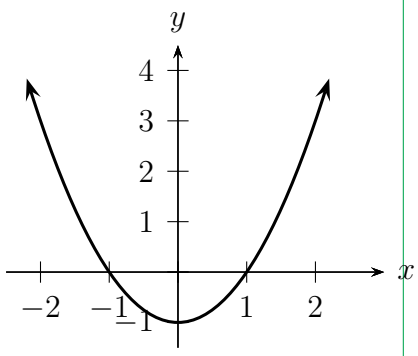


Example 45

Sketching the graph $y = x^2 - 1$.

Listing 9.2 – $y = x^2 - 1$

```
\def\Func{x 2 exp 1 sub}
\begin{psgraph}{->}(0,0)(-2.5,-1.5)(3,4.5){5cm}{4cm}
\psplot[linewidth=1.15pt,arrows=<->]{-2.2}{2.2}{\Func}
\end{psgraph}
```

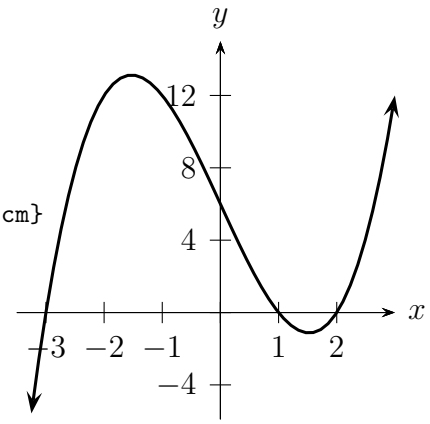


Example 46

Sketching the graph $y = x^3 - 7x + 6$.

Listing 9.3 – $y = x^3 - 7x + 6$

```
\def\Func{x 3 exp -7 x mul add 6 add}
\begin{psgraph}[Dy=4]{->}(0,0)(-3.5,-5.9)(3,15){5cm}{5cm}
\psplot[linewidth=1.15pt,arrows=<->]{-3.25}{3}{\Func}
\end{psgraph}
```



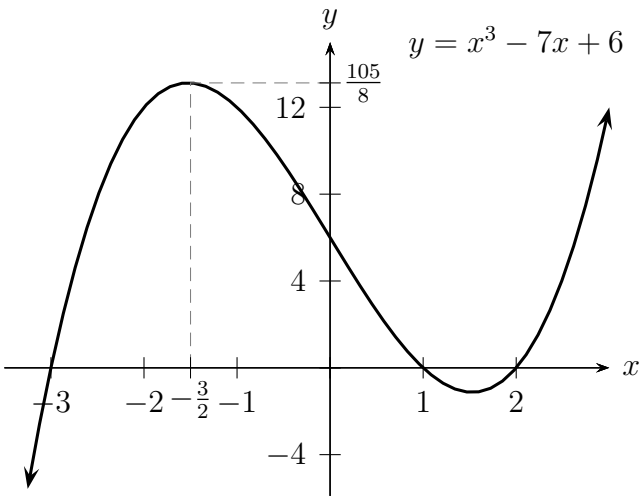
- Using `psgraph` can solve many problems of the vertical axis being too large to fit. In this situation, an absolute height of 5 cm for the entire graph is required, and PSTricks then scales the axes accordingly to fit 5 cm.
- The frequency of the ticks/label can be adjusted by the optional arguments `Dx=1` (one x axis tick/label every unit) and `Dy=4` (one y axis tick/label every 4 units).

Example 47

Sketching the graph $y = x^3 - 7x + 6$, and to find the corresponding y value *programatically* when $x = -\frac{3}{2}$:
(Note: PSTricks can compute the y value programatically – when $x = -\frac{3}{2}$, $y = \frac{105}{8}$)

Listing 9.4 – $y = x^3 - 7x + 6$

```
\def\Func{x 3 exp -7 x mul add 6 add}
\begin{psgraph}[Dy=4]{->}(0,0)(-3.5,-5.9)(3,15){8cm}{6cm}
\psplot[linewidth=1.15pt,arrows=<->]{-3.25}{3}{\Func}
\pnode(2,15){L}
\rput[tc](L){$y = x^3 - 7x + 6$}
\pnode(!3 -2 div /x 3 -2 div def \Func){A}
\pnode(0,0|A){C}
\pnode(A|0,0){D}
\nput{0}{C}{$\frac{105}{8}$}
\nput{-90}{D}{$-\frac{3}{2}$}
\psline[linestyle=dashed,linecolor=gray](D)(A)(C)
\psyTick(13.125){}
\psxTick(-1.5){}
\end{psgraph}
```



Some new commands related to nodes

Line 4: `\pnode(2,15){L}` places an *invisible* node at $(1, 12)$, with name L.

- L can now be used whenever a pair of coordinates is required as an argument!

Line 5: `\rput[c](L){$y = x^3 - 7x + 6$}` places the text $\boxed{y = x^3 - 7x + 6}$ at the node L, alignment centre.

- Other horizontal alignment options: `l` (left), `c` (centre), `r` (right)
- Can combine with vertical alignment options: `t` (top), `b` (bottom), `B` (Baseline), i.e. `\rput[tr](L){$y = x^3 - 7x + 6$}` the text at the node L aligned top and right relative to the node.

Line 6: `\pnode(!3 -2 div /x 3 -2 div def \Func){A}` enters the *PostScript calculation mode* to calculate the coordinates of the node (Under normal circumstances, `\pnode` uses two coordinates, e.g. $(3, 2)$ indicates $x = 3$ and $y = 2$). Under the PostScript calculation mode, i.e. `(! ... )`, the

- first coordinate (x) is calculated from `3 -2 div` ($x = -\frac{3}{2}$)
- second coordinate (y) is calculated from by defining $x = -\frac{3}{2}$, and the substituting into what `\Func` is defined to obtain the corresponding y value, noting that `\Func` was defined in terms of `x`.

Lines 7/8: Another way of specifying coordinates in PSTricks: `\pnode(A|B){C}` takes the x coordinate of node A and y coordinate of node B and places a node at (x_A, y_B) with the node name C.

In this situation,

- `\pnode(0,0|A){C}` gives a node C placed at the x coordinate of $(0, 0)$, y coordinate of A.
- `\pnode(A|0,0){D}` gives a node D placed at the x coordinate of A, y coordinate of $(0, 0)$

Lines 9/10: `\nput{angle}{node}{stuff}` labels nodes by placing `stuff`, with placement `angle`[1] at `node`.

- `\nput{0}{C}{$\frac{105}{8}$}` places $\frac{105}{8}$ at C at a placement angle of 0°relative to where C is.

[1]Mathematical angle with 0° being positive x direction, 90° being positive y direction, 180° being negative x direction and 270° being negative y direction

9.1.3 Trigonometric and Exponential curves

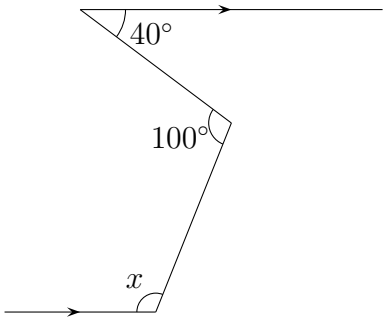
These require the package `pst-math`. The package introduces the following functions:

Function	Example	Operation	Result
`PI`	`2 PI mul`	2π	$6.283185\cdots$
`SIN`	`x SIN`	$\sin(x)$	-1
`COS`	`2 x mul COS`	$\cos(2x)$	0
`TAN`	`PI x mul 4 div TAN`	$\tan\left(\frac{\pi}{4}x\right)$	1
`SEC`	`PI 3 div SEC`	$\sec\left(\frac{\pi}{3}\right)$	2
`COSEC`	`PI 2 div COSEC`	$\operatorname{cosec}\left(\frac{\pi}{2}\right)$	1
`COT`	`PI 6 div COT`	$\cot\left(\frac{\pi}{6}\right)$	$1.73205\cdots$
`ASIN`	`1 2 div ASIN`	$\sin^{-1}\frac{1}{2}(=\frac{\pi}{6})$	$0.52359\cdots$
`ACOS`	`1 2 div ACOS`	$\cos^{-1}\frac{1}{2}(=\frac{\pi}{3})$	$1.04719\cdots$
`ATAN`	`1 3 sqrt div ATAN`	$\tan^{-1}\frac{1}{\sqrt{3}}=\frac{\pi}{6}$	$0.52359\cdots$
`EXP`	`2 EXP`	$\exp(2)=e^2$	$7.389056\cdots$
`ln`	`7 ln`		$1.945910\cdots$
`log`	`100 log`		2
`GAUSS`	`x 0 1 GAUSS`	$f(x)=\frac{1}{\sqrt{2\pi}}\exp\left(\frac{x^2}{2}\right)$	

Table 9.2 – PostScript commands introduced by `pst-math` useful for curve sketching

9.2 Geometric figures

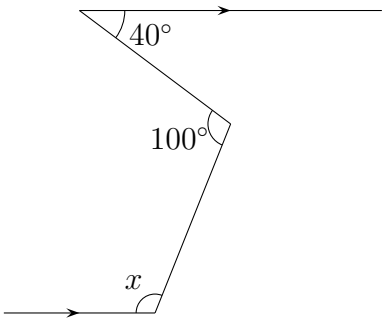
- The `pst-eucl` package comes in very handy.
- Common commands include:
  - `\pstLineAB` to draw lines
  - `\uput` to place something relative to something else.
  - `\pstMarkAngle` and `\pstRightAngle` to draw angle markings.
  - `\pstTranslation` to draw line segments that are parallel/antiparallel.
  - `\nput` to place point names
  - `\pstSegmentMark` to draw a line segment with equal length markings. See Example 49 on page 57
- Common options:
  - `ArrowInside` for `\pstLineAB`
  - `RightAngleSize` for `\pstRightAngle`
  - `MarkAngleRadius` and `LabelSep` for `\pstMarkAngle`
  - `DistCoef` for `\pstTranslation` to stretch/shrink the parallel lines
  - `SegmentSymbol` and `MarkAngle` for `\pstSegmentMark`



Example 48

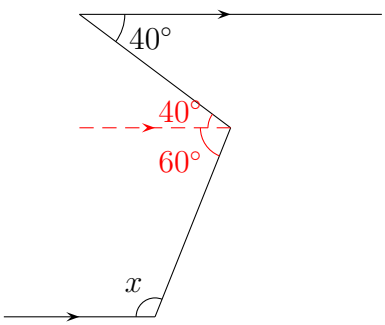
Listing 9.5 – Simple parallel lines configuration geometrical diagram

```
\psset{PointName=none,PointSymbol=none}
\begin{pspicture}(0,0)(5,4)
\pstGeonode(0,0){O}(2,0|O){A}(3,2.5){B}(1,4){C}(5,4){D}
\pstLineAB[ArrowInside=->]{O}{A}
\pstLineAB{A}{B}
\pstLineAB{B}{C}
\pstLineAB[ArrowInside=->]{C}{D}
\pstMarkAngle[MarkAngleRadius=0.25,LabelSep=0.5]{B}{A}{O}{$x$}
\pstMarkAngle[MarkAngleRadius=0.3,LabelSep=0.7]{C}{B}{A}{$100\degree$}
\pstMarkAngle[MarkAngleRadius=0.6,LabelSep=1]{B}{C}{D}{$40\degree$}
\end{pspicture}
```



Listing 9.6 – Simple parallel lines configuration geometrical diagram - corresponding solution

```
\psset{PointName=none,PointSymbol=none}
\begin{pspicture}(0,0)(5,4)
\pstGeonode(0,0){O}(2,0|O){A}(3,2.5){B}(1,4){C}(5,4){D}
\pstLineAB[ArrowInside=->]{O}{A}
\pstLineAB{A}{B}
\pstLineAB{B}{C}
\pstLineAB[ArrowInside=->]{C}{D}
\pstMarkAngle[MarkAngleRadius=0.25,LabelSep=0.5]{B}{A}{O}{$x$}
%\pstMarkAngle[MarkAngleRadius=0.3,LabelSep=0.7]{C}{B}{A}{$100\degree$}
\pstMarkAngle[MarkAngleRadius=0.6,LabelSep=1]{B}{C}{D}{$40\degree$}
\psset{linecolor=red}
\pstTranslation{A}{O}{B}[B']
\pstLineAB[ArrowInside=->,linestyle=dashed]{B'}{B}
\pstMarkAngle[MarkAngleRadius=0.3,LabelSep=0.7]{C}{B}{B'}{\red $40\degree$}
\pstMarkAngle[MarkAngleRadius=0.4,LabelSep=0.8]{B'}{B}{A}{\red $60\degree$}
\end{pspicture}
```

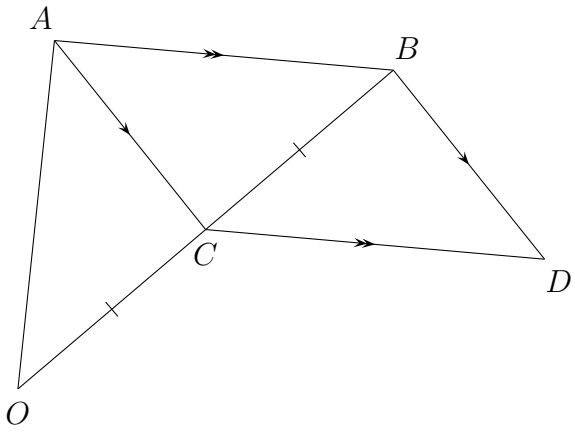


Example 49

Listing 9.7 – Vector geometry question

```
\paragraph{[2020 NEAP Ext 1 Trial Q7]} The position vectors of points $A$ and $B$
are $\vb{a}$ and $\vb{b}$ respectively.
Point $C$ is the midpoint of $OB$ and point $D$ is such that $ABCD$ is a parallelogram.
\begin{center}
\begin{pspicture}[showgrid=false](0,0)(8,5)
\pnode(1,5){A}
%\pnode(4,4.75){B}
\uput{4.5\psxunit}[-5](A){\pnode{B}}
\pnode(3,2.5){C}
\pstTranslation[PointName=none,PointSymbol=none]{A}{B}{C}[D]
\pstTranslation[PointName=none,PointSymbol=none]{B}{C}{C}[O]
\pstLineAB{O}{A}
\pstLineAB[ArrowInside=->>]{A}{B}
\pstLineAB[ArrowInside=->>]{C}{D}
\pstLineAB[ArrowInside=->]{A}{C}
\pstLineAB[ArrowInside=->]{B}{D}
\pstSegmentMark[SegmentSymbol=MarkHash,MarkAngle=90]{O}{C}
\pstSegmentMark[SegmentSymbol=MarkHash,MarkAngle=90]{C}{B}
\nput{120}{A}{$A$}
\nput{60}{B}{$B$}
\nput{-90}{C}{$C$}
\nput{-60}{D}{$D$}
\nput{-90}{O}{$O$}
\end{pspicture}
\end{center}
Which of the following is the position vector of $D$?
\begin{multicols}{4}
\begin{enumerate}[label=(\Alph*)]
\item    $\frac{3}{2} \vb{b} + \vb{a}$
\item    $\frac{3}{2} \vb{b} - \vb{a}$
\item    $\frac{1}{2} \vb{b} - \frac{1}{2}\vb{a}$
\item    $\frac{1}{2} \vb{b} - \vb{a}$
\end{enumerate}
\end{multicols}
```

[2020 NEAP Ext 1 Trial Q7] The position vectors of points A and B are $\underset{\sim}{a}$ and $\underset{\sim}{b}$ respectively. Point C is the midpoint of OB and point D is such that $ABCD$ is a parallelogram.



Which of the following is the position vector of D?

(A) $\frac{3}{2}\underset{\sim}{b} + \underset{\sim}{a}$ (B) $\frac{3}{2}\underset{\sim}{b} - \underset{\sim}{a}$ (C) $\frac{1}{2}\underset{\sim}{b} - \frac{1}{2}\underset{\sim}{a}$ (D) $\frac{1}{2}\underset{\sim}{b} - \underset{\sim}{a}$

9.3 Diagrams

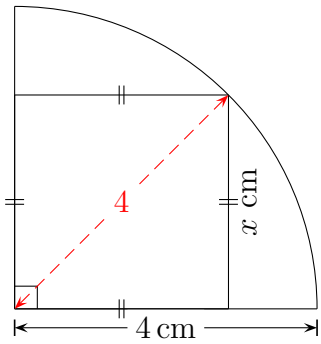
Miscellaneous diagrams.

9.3.1 General 2D diagrams

Example 50

Listing 9.8 – Quarter circle

```
\psset{PointName=none,PointSymbol=none}
\begin{pspicture}(0,-0.5)(4,4)
\pstGeonode(0,0){O}(4,0){A}(0,4){B}
\pstArcOAB{O}{A}{B}
\pstLineAB{O}{A}
\pstLineAB{O}{B}
\pstLineAB[offset=-0.25cm,arrows=|<*->|*]{O}{A} \ncput*{\np[cm]{4}}
\pnode(4;45){X}
\pstGeonode(X|0,0){X1}(0,0|X){X2}
\pstSegmentMark[SegmentSymbol=MarkHashh,MarkAngle=90]{X1}{X} \nbput[nrot=:U]{$x$ cm}
\pstSegmentMark[SegmentSymbol=MarkHashh,MarkAngle=90]{X2}{X}
\pstSegmentMark[SegmentSymbol=MarkHashh,MarkAngle=90]{O}{X1}
\pstSegmentMark[SegmentSymbol=MarkHashh,MarkAngle=90]{O}{X2}
\pstRightAngle[RightAngleSize=0.3]{B}{O}{A}
\psset{linecolor=red}
\pstLineAB[arrows=<->,linestyle=dashed]{O}{X} \ncput*{\red 4}
\end{pspicture}
```

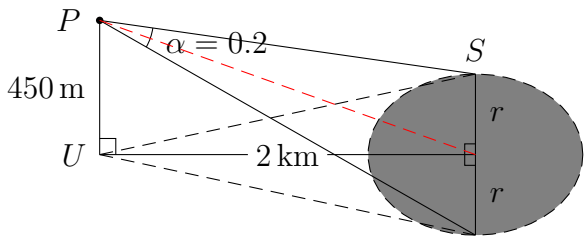


9.3.2 General 3D diagrams

Example 51

Listing 9.9 – 3D Trig

```
\psset{unit=0.71cm}
\begin{pspicture}(-1,-2)(10,2.75)
\pnode(0,0){O}
\pstGeonode[PosAngle={180,0}](0,2.5){P}(7,0){T}
\pstEllipse[linestyle=dashed,fillstyle=solid,fillcolor=gray,opacity=0.5](T)(2,1.5)[0][360]
\pstLineAB{O}{T} \ncput*{\np[km]{2}}
\pstLineAB{O}{P} \naput{\np[m]{450}}
\pstRightAngle[RightAngleSize=0.3]{T}{O}{P}
\pnode(T|0,1.5){T1}
\pnode(T|0,-1.5){T2}
\pstLineAB[linestyle=dashed]{O}{T1}
\pstLineAB[linestyle=dashed]{O}{T2}
\pstLineAB{P}{T1}
\pstLineAB{P}{T2}
\pstLineAB{T2}{T} \nbput{$r$}
\pstLineAB{T}{T1} \nbput{$r$}
\psset{RightAngleSize=0.2}
\pstRightAngle{O}{T}{T1}
\pstRightAngle{O}{T}{T2}
\pstMarkAngle[MarkAngleRadius=1,LabelSep=1.3,LabelRefPt=l]{T2}{P}{T1}{$\alpha = 0.2$}
\psset{linecolor=red}
\pstLineAB[linestyle=dashed]{P}{T}
\nput{180}{O}{$U$}
\nput{90}{T1}{$S$}
\end{pspicture}
```

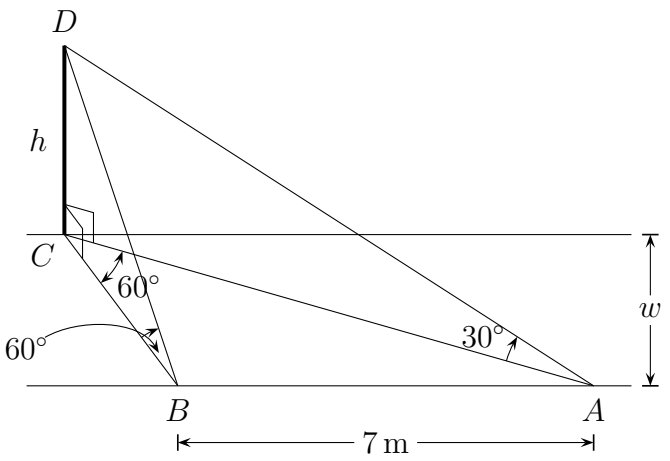


- Drawing right angles in 3D trig questions can be tricky due to perspective. Another example will be provided.
- `\pstEllipse` is able to draw ellipses with a certain centre and semi-major/semi-minor axis lengths, within a particular angle range. See the `pst-eucl` documentation for further details

Example 52

Listing 9.10 – 3D Trig with right angles

```
\begin{pspicture}[showgrid=false](0,-1.25)(9,5)
\pnode(0,0){L}
\pnode(8,0){M}
\pnode(0,2){N}
\pnode(M|N){P}
\pstLineAB{L}{M}
\pstLineAB{N}{P}
\pnode(0.5,0|N){C}
\pnode(C|0,4.5){D}
\pstLineAB[linewidth=1.5pt]{C}{D} \naput{$h$}
\pnode(2,0){B}
\pnode(7.5,0){A}
\pstLineAB{C}{B}
\pstLineAB{C}{A}
\pstLineAB{D}{B}
\pstLineAB{D}{A}
\pstLineAB[offset=-0.75,arrows=|<*->|*]{B}{A} \ncput*{\np[m]{7}}
\nput{90}{D}{$D$}
\nput{-135}{C}{$C$}
\nput{-90}{B}{$B$}
\nput{-90}{A}{$A$}
\pstMarkAngle[MarkAngleRadius=0.8,LabelSep=1.2,arrows=<->]{B}{C}{A}{$60\degree$}
\psset{PointName=none,PointSymbol=none}
\uput{0.4\psxunit}[(B)](C){\pnode{C1}}
\uput{0.4\psxunit}[(D)](C){\pnode{C2}}
\pstTranslation{C}{C1}{C2}[C3]
    \psline(C2)(C3)(C1)
\uput{0.4\psxunit}[(A)](C){\pnode{C4}}
\pstTranslation{C}{C4}{C2}[C5]
    \psline(C2)(C5)(C4)
\pstMarkAngle[MarkAngleRadius=0.8,LabelSep=1.2,arrows=<-]{D}{B}{C}{}
\pstMarkAngle[MarkAngleRadius=1.2,LabelSep=1.6,arrows=<-]{D}{A}{C}{$30\degree$}
\rput(0,0.5){\rnode{DBC}{$60\degree$}}
\nccurve[angleA=30,angleB=120,arrows=->,nodesepB=0.5\psxunit]{DBC}{B}
\pstLineAB[arrows=|<*->|*,offset=-0.25]{M}{P} \ncput*{$w$}
\end{pspicture}
```

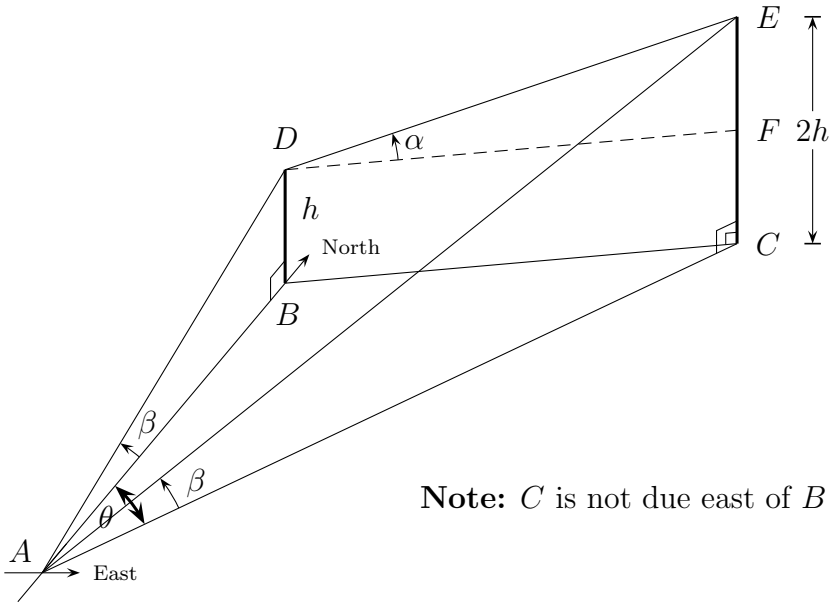


Example 53

Listing 9.11 – Another 3D Trig example with right angles

```
\psset{PointSymbol=none}
\begin{pspicture}[showgrid=false](-1,-1)(11,8)
\pstGeonode[PosAngle=135,PointSymbol=none](0,0){A}
\pnode(-0.5,0){A1}
\pnode(0.5,0){A2}   \nput{0}{A2}{\scriptsize East}
\pstLineAB[arrows=->]{A1}{A2}
\pnode(0.5;50){B2}
\pstTranslation[PointName=,PointSymbol=none]{B2}{A}{A}[B1]
\pstLineAB{B1}{B2}
\uput{5\psxunit}[(B2)](A){\pstGeonode[PosAngle=-85]{B}}
\pstLineAB{A}{B}
%%% Draw
\pstTranslation[PointName=,PointSymbol=none]{A}{B2}{B}[B3]
\pstLineAB[arrows=->]{B}{B3}
\nput{20}{B3}{\scriptsize North}
\uput{1.5\psxunit}[90](B){\pstGeonode[PosAngle=90]{D}}
\pstLineAB[linewidth=1.15pt]{B}{D} \nbput[npos=0.66]{$h$}
\pstLineAB{A}{D}
\pstMarkAngle[MarkAngleRadius=2,LabelSep=2.4,LabelRefPt=c,arrows=->]{B}{A}{D}{$\beta$}
%%% Draw right angle ABD
\uput{0.3\psxunit}[(A)](B){\pnode{A3}}
\uput{0.3\psxunit}[(D)](B){\pnode{D3}}
\pstTranslation[PointName=,PointSymbol=none]{B}{A3}{D3}[B4]
\psline(D3)(B4)(A3)
%%% Draw CE
\uput{6\psxunit}[5](B){\pstGeonode{C}}
\pstLineAB{B}{C}
\pstTranslation{B}{D}{C}[F]
\pstTranslation{C}{F}{F}[E]
\pstLineAB[linewidth=1.15pt]{C}{E}
\pstLineAB[offset=-1,arrows=|<*->|*]{C}{E}
\ncput*{$2h$}
%%% Finish off rectangle
\pstLineAB[linestyle=dashed]{D}{F}
\pstLineAB{D}{E}
\pstMarkAngle[MarkAngleRadius=1.5,LabelSep=1.75,LabelRefPt=c,arrows=->]{F}{D}{E}{$\alpha$}
%%% Draw right angles
\uput{0.15\psxunit}[(F)](C){\pnode{C1}}
\uput{0.15\psxunit}[(B)](C){\pnode{C3}}
\pstTranslation[PointName=none,PointSymbol=none]{C}{C1}{C3}[C2]
\psline(C1)(C2)(C3)
%%% Finish AC
\pstLineAB{A}{C}
\pstMarkAngle[MarkAngleRadius=2,LabelSep=2.25,LabelRefPt=l,arrows=->]{C}{A}{E}{$\beta$}
\pstLineAB{A}{E}
\uput{0.3\psxunit}[(A)](C){\pnode{C10}}
\uput{0.3\psxunit}[(F)](C){\pnode{C30}}
\pstTranslation[PointName=none,PointSymbol=none]{C}{C10}{C30}[C20]
\psline(C10)(C20)(C30)
%%% Draw theta
\pstMarkAngle[MarkAngleRadius=1.5,LabelSep=1.2,LabelRefPt=r,arrows=<->,%
    linestyle=dashed,linewidth=1.15pt]{C}{A}{B}{$\theta$}
\rput[l](5,1){\textbf{Note:} $C$ is not due east of $B$}
\end{pspicture}
```

Example 53 on the previous page

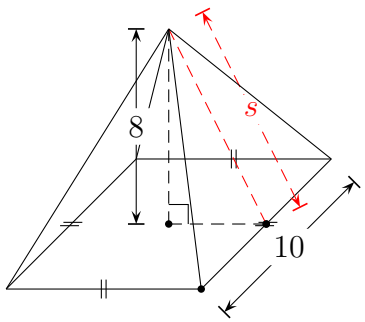


Note: C is not due east of B

Example 54

Listing 9.12 – Triangular pyramid

```
\psset{unit=0.86cm}
\begin{pspicture}[showgrid=false](-3,-1.5)(3,3.5)
\pnode(0,0){O}
\pnode(-1,1){A}
\pnode(2,0|A){B}
\pnode(-3,-1){C}
\pstTranslation[PointName=none]{A}{B}{C}[D]
\pstSegmentMark[SegmentSymbol=MarkHashh,MarkAngle=90]{A}{B}
\pstSegmentMark[SegmentSymbol=MarkHashh,MarkAngle=135]{C}{A}
\pstSegmentMark[SegmentSymbol=MarkHashh,MarkAngle=135]{D}{B}
\pstLineAB[offset=-0.5,arrows=|<*->|*]{D}{B}
    \ncput*{$10$}
\pstSegmentMark[SegmentSymbol=MarkHashh,MarkAngle=90]{C}{D}
\pstMiddleAB[PointName=none]{A}{D}{M1}
\pnode(M1|0,3){E}
\pstMiddleAB[PointName=none]{D}{B}{Z}
\pstLineAB[linestyle=dashed,linecolor=red]{Z}{E}
\pstLineAB[linestyle=dashed,linecolor=red,offset=-0.5cm,arrows=|<*->|*]{Z}{E}
    \ncput*{\red $s$}
\pstLineAB{A}{E}
\pstLineAB{B}{E}
\pstLineAB{C}{E}
\pstLineAB{D}{E}
\pstLineAB[linestyle=dashed]{M1}{E}
\pstLineAB[arrows=|<*->|*,offset=0.5]{M1}{E}
    \ncput*{8}
\pstMiddleAB[PointName=none]{D}{B}{M2}
\pstLineAB[linestyle=dashed]{M1}{M2}
\pstRightAngle[RightAngleSize=0.3]{E}{M1}{M2}
\end{pspicture}
```



- `\pstMiddleAB` finds the midpoint between A and B.

9.3.3 Probability or factor tree

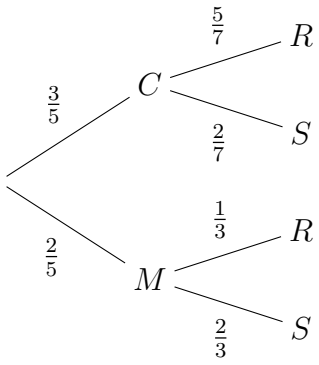
- The `pst-tree` package should be loaded.
- The `unit`, `treesep` and `nodesep` parameters may need tweaking.

Example 55

Listing 9.13 – Probability tree

```
\usepackage{pst-tree} % in the preamble

\psset{unit=1.5cm,treesep=1cm,nodesep=3pt}
\pstree[treemode=R,]{\Tr*{}}{ \large
    \pstree{\Tr*{$C$} \naput*{$\frac{3}{5}$}}{
    \Tr*{$R$}       \naput*{$\frac{5}{7}$}
    \Tr*{$S$}       \nbput*{$\frac{2}{7}$}
}
\pstree{\Tr*{$M$} \nbput*{$\frac{2}{5}$}}{
    \Tr*{$R$}       \naput*{$\frac{1}{3}$}
    \Tr*{$S$}       \nbput*{$\frac{2}{3}$}
    }
}
```

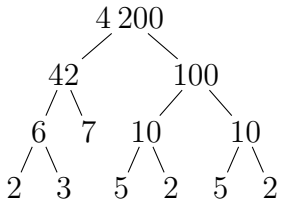


Example 56

Listing 9.14 – Factor tree

```
\usepackage{pst-tree} % in the preamble

\psset{treesep=0.45cm,nodesep=2pt,levelsep=0.75cm}
\pstree[treemode=D]{\Tr*{\np{4200}}}{
    \pstree{\Tr*{42}}{
        \pstree{\Tr*{6}}{
            \Tr*{2}
            \Tr*{3}
        }
        \Tr*{7}
    }
    \pstree{\Tr*{100}}{
        \pstree{\Tr*{10}}{
            \Tr*{5}
            \Tr*{2}
    }
    \pstree{\Tr*{10}}{
        \Tr*{5}
        \Tr*{2}
        }
    }
}
```



References

Patel, S. K. (2004). *Maths Extension 2* (2nd ed.). Pascal Press.